analogy, are found to be in agreement with the experiment. Although the present technique requires experimental data, in the form of the vortex core locations, the model does account for the previously ignored mass entrainment of the vortex core.

References

[1] Coe, P. L., Chambers, J. R., and Letko, W., "Asymmetric Lateral-Directional Characteristics of Pointed Bodies of Revolution at High Angles of Attack," TN D-7095, Nov. 1972, NASA.

[2] Brown, C. E. and Michael, W. H., "On Slender Delta Wings With Leading-Edge Separation," TN 3430, 1955, NACA.

[3] Mangler, K. W. and Smith, J. H. B., "A Theory of the Flow Past Slender Delta Wings With Leading Edge Separation," *Proceedings of the Royal Society,* A-251, 1959, pp. 200–217.

[4] Gersten, K., "Calculation of Non-Linear Aerodynamic Stability Derivatives of Aeroplanes," AGARD 342, 1961.

[5] Smith, J. H. B., "Improved Calculations of Leading Edge Separation From Slender Delta Wings," TR 66070, March 1966, Royal Aircraft Establishment, Farnborough, England.

[6] Polhamus, E. C., "A Concept of the Vortex Lift of Sharp Edge Delta Wings Based on a Leading-Edge-Suction Analogy," TN D-3767, 1966, NASA.

[7] Coe, P. L., "Stationary Vortices Behind a Flat Plate Normal to the Freestream in Incompressible Flow," *AIAA Journal,* Vol. 10, No. 12, Dec. 1972, p. 1701.

[8] Werle', H., "Tourbillons D'Ailes Minces Très Élancées," *La Recherche Aérospatiale,* No. 109, Nov.–Dec. 1965, pp. 3–12.

[9] Peckham, D. H., "Low-Speed Wind-Tunnel Tests on a Series of Uncambered Slender Pointed Wings With Sharp Edges," RM 3186, 1961, Aeronautical Research Council, London, England.

[10] Fink, P. T., "Wind-Tunnel Tests on a Slender Delta Wing at High Incidence," *Zeitschrift für Flugwissenschaften,* Jahrg. 4, Heft 7, July 1956, pp. 247–249.

Determination of Bending Influence Coefficients from Bending Slope Data

Henry E. Fettis*
Technology, Inc., Dayton Ohio

Bending influence coefficients for a cantilever beam are defined as follows: 1) $H_h(x, \bar{x})$ = vertical deflection at "x" due to unit force at "$\bar{x}$" (in./lb); 2) $H_\theta(x, \bar{x})$ = vertical deflection at "x" due to unit moment at "$\bar{x}$" (in./in. lb); 3) $\theta_h(x, \bar{x})$ = angular deflection at "x" due to unit force at "$\bar{x}$" (rad/lb); 4) $\theta_\theta(x, \bar{x})$ = angular deflection at "x" due to unit moment at "$\bar{x}$" (rad/in. lb). Direct determination of these quantities would normally require several sets of measurements for each separate loading. It will be shown that all of these quantities can be found either directly or by simple quadrature from a single set of measurements, namely, the angular deflection distribution due to a unit moment applied at the tip.

The analytical expressions for the various influence coefficients in terms of bending stiffness, EI, are

$$\begin{cases} H_h(x, \bar{x}) = \displaystyle\int_0^x \frac{(x-\xi)(\bar{x}-\xi)}{EI(\xi)} d\xi, & x \leqq \bar{x} \\ H_h(x, \bar{x}) = \displaystyle\int_0^{\bar{x}} \frac{(x-\xi)(\bar{x}-\xi)}{EI(\xi)} d\xi, & x \geqq \bar{x} \end{cases}$$

$$\begin{cases} H_\theta(x, \bar{x}) = \displaystyle\int_0^x \frac{(x-\xi)}{EI(\xi)} d\xi, & x \leqq \bar{x} \\ H_\theta(x, \bar{x}) = \displaystyle\int_0^{\bar{x}} \frac{(x-\xi)}{EI(\xi)} d\xi, & x \geqq \bar{x} \end{cases}$$

$$\begin{cases} \theta_h(x, \bar{x}) = \displaystyle\int_0^x \frac{(\bar{x}-\xi)}{EI(\xi)} d\xi, & x \leqq \bar{x} \\ \theta_h(x, \bar{x}) = \displaystyle\int_0^{\bar{x}} \frac{(\bar{x}-\xi)}{EI(\xi)} d\xi, & x \geqq \bar{x} \end{cases}$$

$$\begin{cases} \theta_\theta(x, \bar{x}) = \displaystyle\int_0^x \frac{d\xi}{EI(\xi)}, & x \leqq \bar{x} \\ \theta_\theta(x, \bar{x}) = \displaystyle\int_0^{\bar{x}} \frac{d\xi}{EI(\xi)}, & x \geqq \bar{x} \end{cases}$$

From these equations it is seen that

$$H_h(x, \bar{x}) = H_h(\bar{x}, x)$$
$$H_\theta(x, \bar{x}) = \theta_h(\bar{x}, x)$$
$$\theta_\theta(x, \bar{x}) = \theta_\theta(\bar{x}, x)$$

If $\phi(x)$ is the angular deflection at x due to a unit moment at the tip, then

$$\phi(x) = \int_0^x \frac{d\xi}{EI(\xi)}$$

Hence

$$\frac{1}{EI(x)} = \frac{d\phi}{dx}$$

Inserting the expression for $(1/EI)$ in the equations for the influence coefficients and simplifying, we obtain

$$\begin{cases} H_h(x, \bar{x}) = (x+\bar{x}) \displaystyle\int_0^x \phi(\xi)\, d\xi - 2 \int_0^x \phi(\xi)\xi\, d\xi & x \leqq \bar{x} \\ H_h(x, \bar{x}) = (x+\bar{x}) \displaystyle\int_0^{\bar{x}} \phi(\xi)\, d\xi - 2 \int_0^{\bar{x}} \phi(\xi)\xi\, d\xi & x \geqq \bar{x} \end{cases}$$

$$\begin{cases} H_\theta(x, \bar{x}) = \displaystyle\int_0^x \phi(\xi)\, d\xi & x \leqq \bar{x} \\ H_\theta(x, \bar{x}) = (x-\bar{x})\phi(\bar{x}) + \displaystyle\int_0^{\bar{x}} \phi(\xi)\, d\xi & x \geqq \bar{x} \end{cases}$$

$$\begin{cases} \theta_h(x, \bar{x}) = (\bar{x}-x)\phi(x) + \displaystyle\int_0^x \phi(\xi)\, d\xi & x \leqq \bar{x} \\ \theta_h(x, \bar{x}) = \displaystyle\int_0^{\bar{x}} \phi(\xi)\, d\xi & x \geqq \bar{x} \end{cases}$$

$$\begin{cases} \theta_\theta(x, \bar{x}) = \phi(x) & x \leqq \bar{x} \\ \theta_\theta(x, \bar{x}) = \phi(\bar{x}) & x \geqq \bar{x} \end{cases}$$

Received July 9, 1973.

Index category: Aircraft Vibration.

* Consultant in Mathematics, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio.

Stiffness of Orthotropic Materials and Laminated Fiber-Reinforced Composites

Robert M. Jones*
SMU Institute of Technology, Dallas, Texas

Introduction

Chu[1] considered a laminated fiber-reinforced composite with 40% of the fibers aligned with the 1-axis in Fig. 1 and the remaining 60% at ±45° to the 1-axis. His description of the laminate unfortunately did not include the number of layers and the stacking sequence. He was perplexed by the observation that the Young's modulus in an x-direction (E_x) other than the 1-direction was larger than in the 1-direction (E_1).† He concluded

Received July 12, 1973. This work was partially supported under Air Force Office of Scientific Research Grant 73-2532.

Index categories: Properties of Materials; Structural Composite Materials (Including Coatings); Structural Static Analysis.

* Associate Professor of Solid Mechanics. Associate Fellow AIAA.

† See Ref. 2 for notation.